Lecture slides for this course
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Deep Learning

Chapter 2 Building Neural Network from Scratch

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Chapter 2: Building Neural Network from Scratch

1. Shallow neural network
2. Deep neural network
3. Building neural network: step-by-step (modulation)
4. Regularization
5. Dropout
6. Batch Normalization
7. Optimizers
8. Hyper-parameters
9. Practice

Previous Lecture Overview

Chapter 1: Course Infor & Programming review - week 1

1. Course introduction and grades
2. History of Deep learning
3. Deep learning applications

Chapter 2: Building Neural Network from Scratch – week 2-7

1. Shallow neural network - week 2
2. Deep neural network - week 3
3. Building neural network: step-by-step (modulation) - week 3
4. Regularization - week 4
5. Dropout - week 4
6. Batch Normalization - week 5
7. Optimizers - week 6
8. Hyper-parameters - week 7
9. Practice- week

Midterm

Chapter 3: Convolutional Neural Network - week 8-10

1. Convolutional operator
2. History of CNN
3. Deep Convolutional Models
4. Layers in CNN
5. Applications of CNN
6. Practice

Midterm summary

Chapter 4: TensorFlow Library- week 11-13

1. Introduction to TensorFlow
2. Building a deep neural network with TensorFlow
3. Applications
4. Practice

Chapter 5: Recurrent Neural Network week 14-15

1. Unfolding Computational Graphs
2. Building a Recurrent Neural Networks
3. Long Short-Term Memory
4. Vision with Language Processing
5. Application of RNN
6. Practice

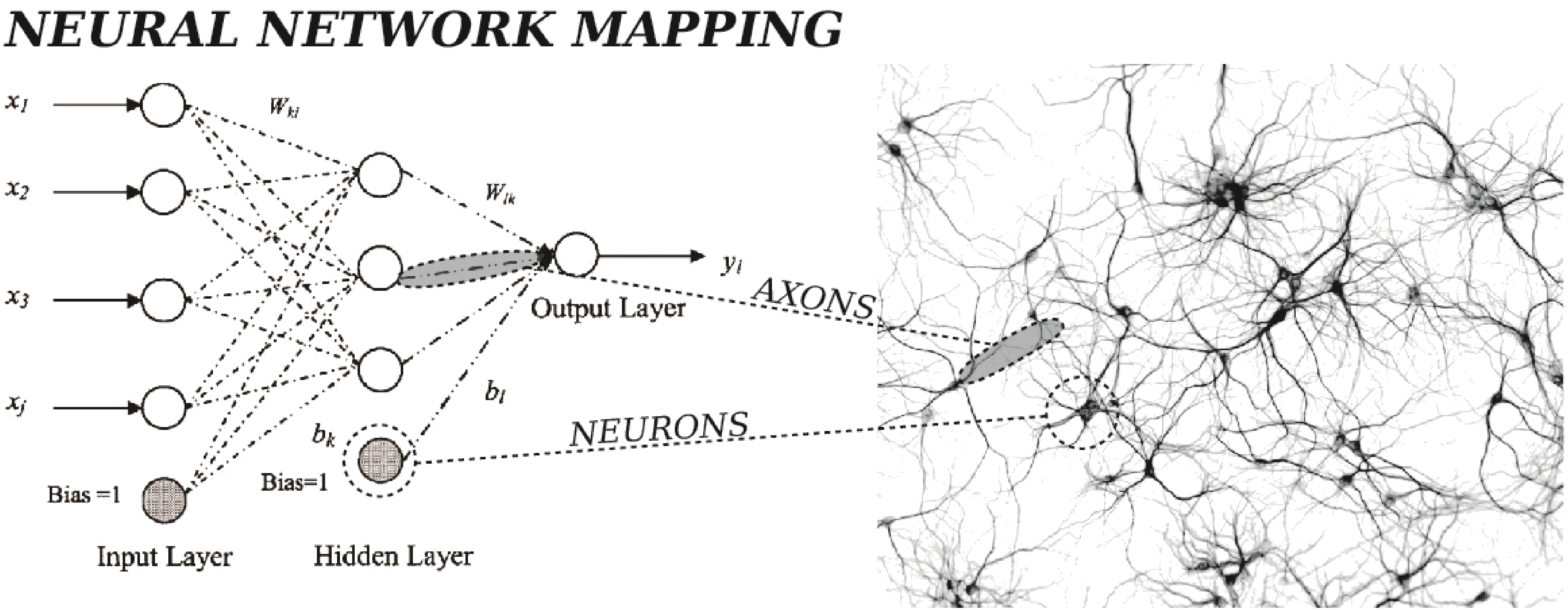
Previous Lecture Overview

Basic of Neural Network

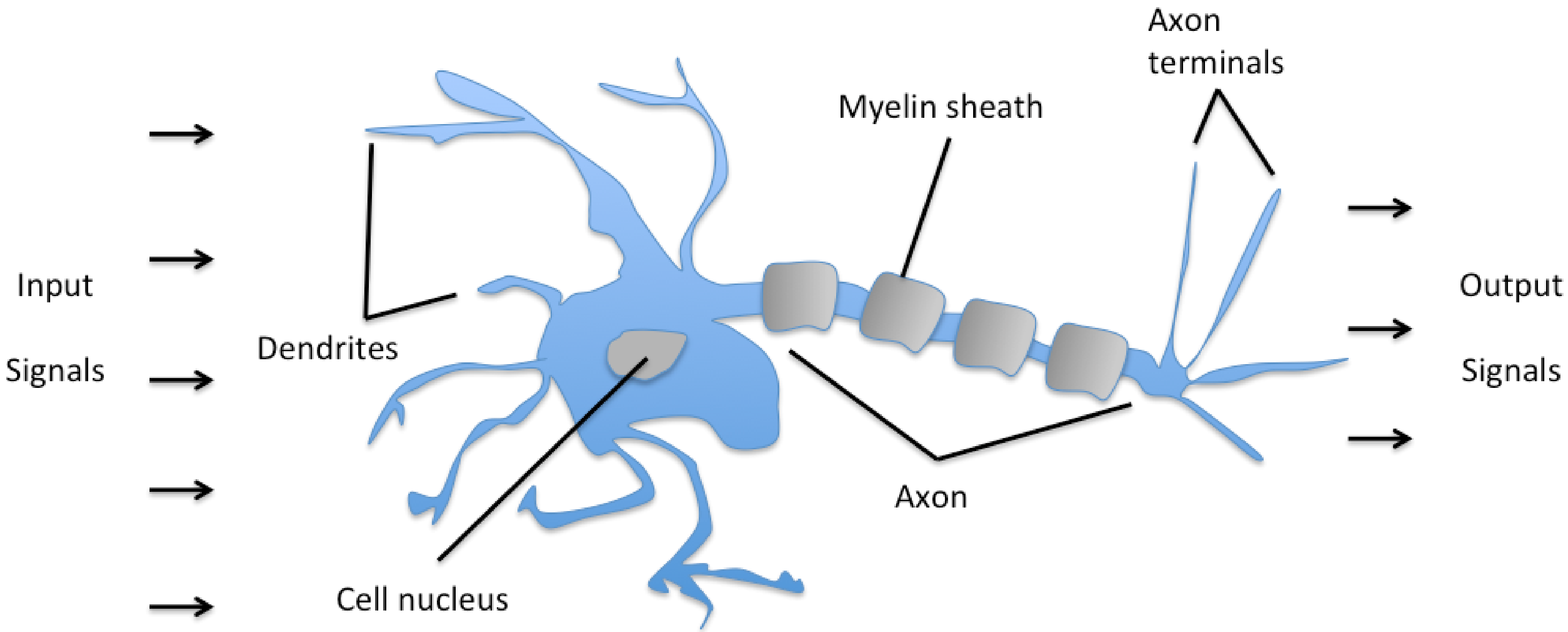
- The Perceptron and its Learning Rule (Frank Rosenblatt, 1957)
- Adaptive Linear Neuron and Delta Rule (Widrow & Hoff, 1960)
- Logistic Regression and Gradient Descent

Previous Lecture Overview

Biologically inspired (akin to the neurons in a brain)



Artificial Neurons and the McCulloch-Pitts Model (1943)

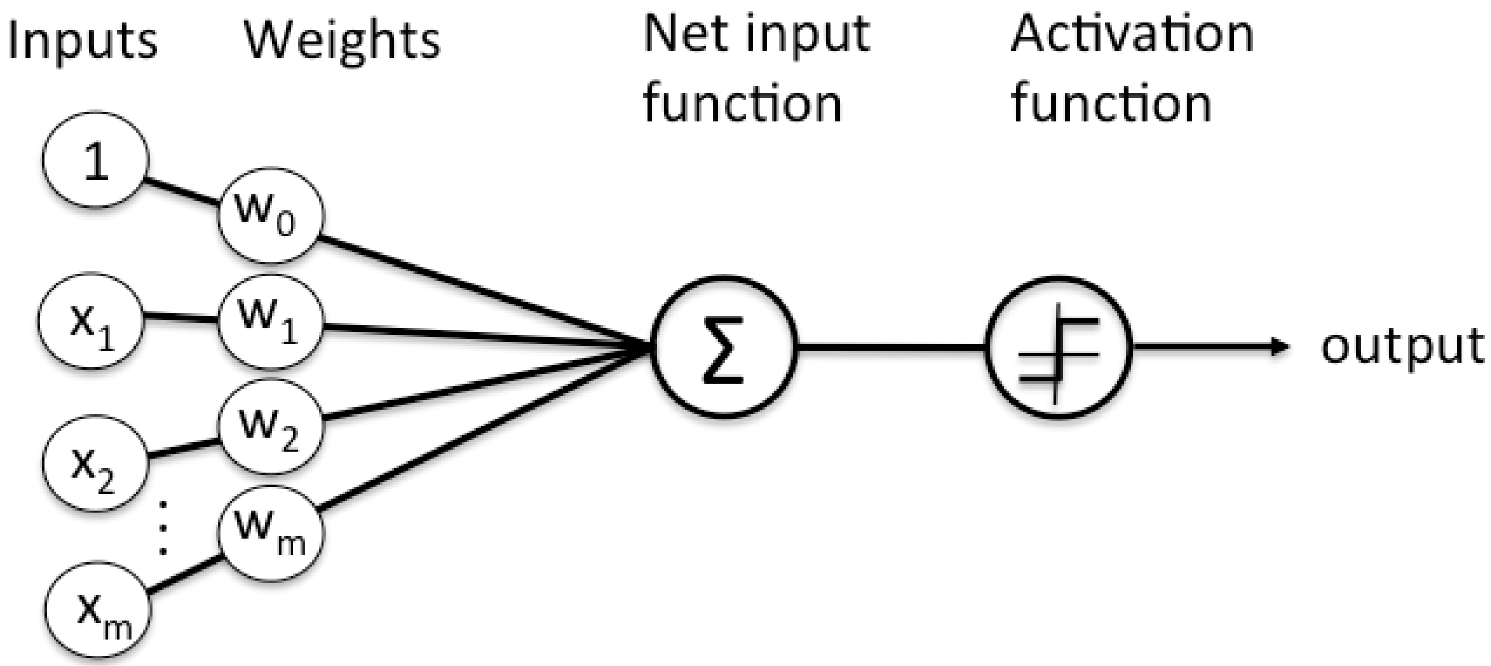


Schematic of a biological neuron.

W. S. McCulloch and W. Pitts. A logical calculus of the ideas immanent in nervous activity. The bulletin of mathematical biophysics, 5(4):115–133, 1943.

Previous Lecture Overview

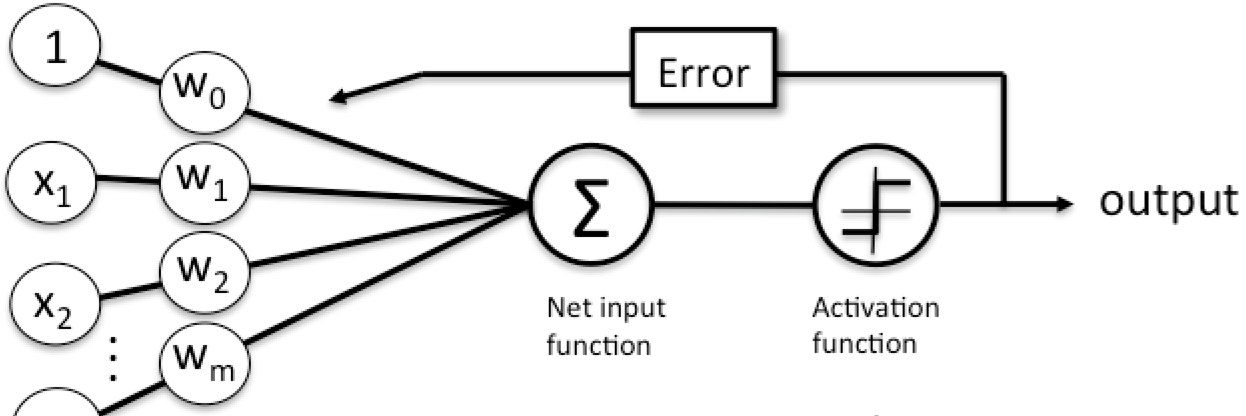
Frank Rosenblatt’s Perceptron (1957)



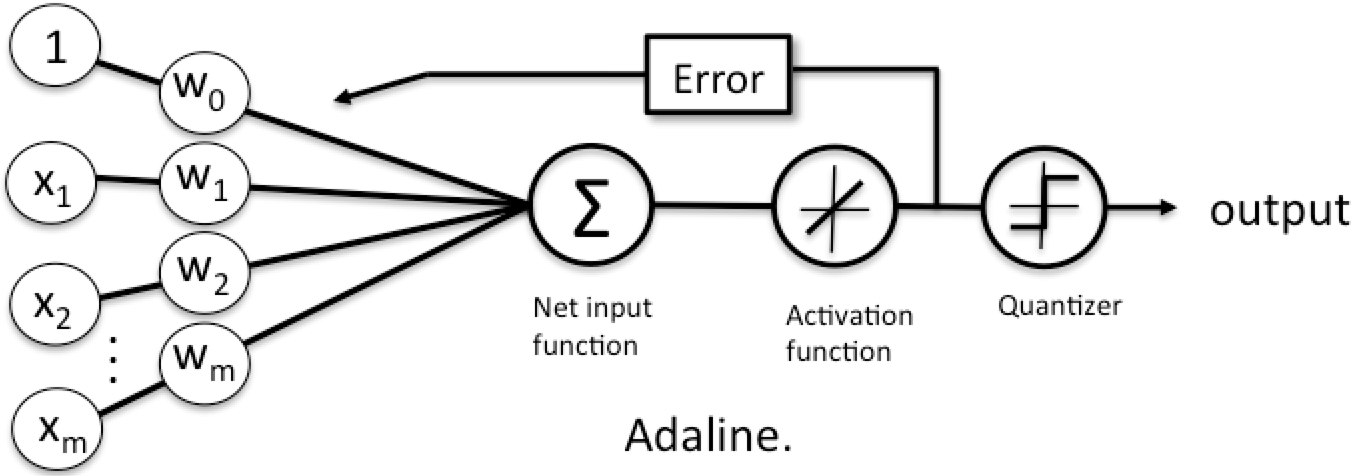
Schematic of Rosenblatt’s perceptron.

Previous Lecture Overview

Adaptive Linear Neurons and the Delta Rule (1960)



Perceptron rule.

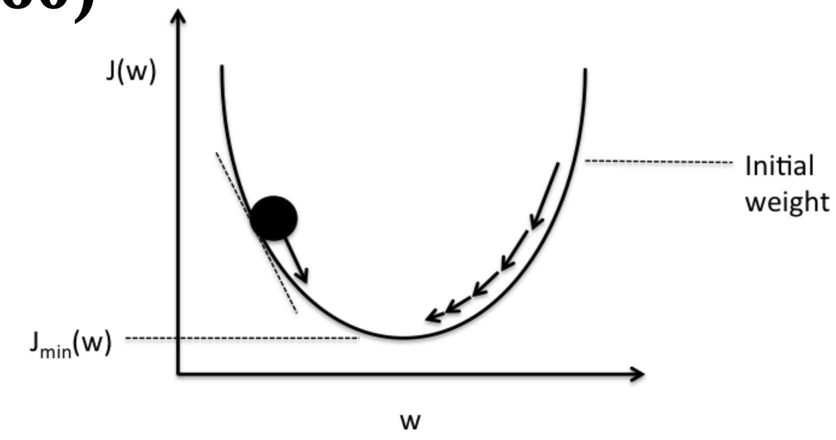


Adaline.

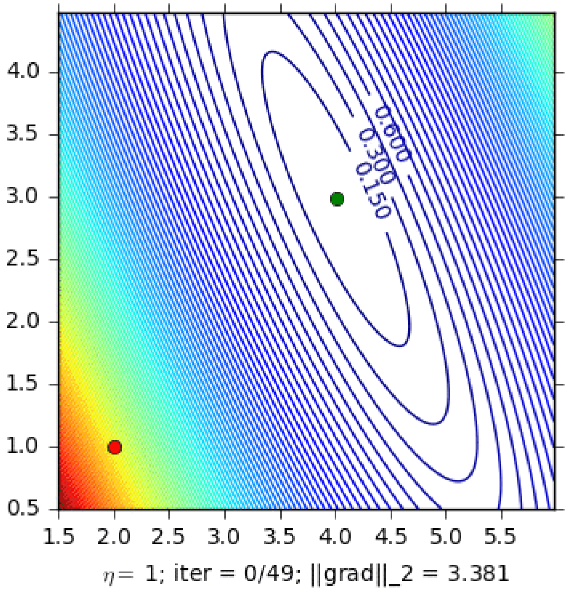
Previous Lecture Overview

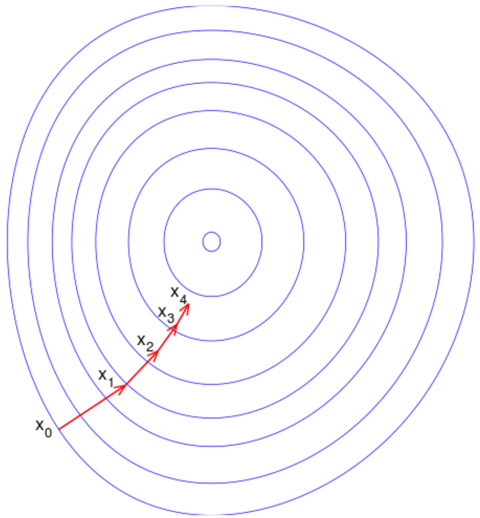
Adaptive Linear Neurons and the Delta Rule (1960)

- **Gradient Descent**
  - A first-order iterative optimization algorithm for finding the minimum of a function
  - Take steps proportional to the **negative of the gradient** of the function at the current point



Schematic of gradient descent.





Previous Lecture Overview

Adaptive Linear Neurons and the Delta Rule (1960)

- Cost function: sum of squared errors (SSE)
  - $J(w) = \frac{1}{2}\sum_i (y'^{(i)} - y^{(i)})^2$
- To minimize SSE, we can use "gradient descent"
- A step in the opposite direction of gradient

$$\Delta w = -\alpha \nabla J(w)$$

where α is the learning rate, $0 < \alpha < 1$

- Thus, we need to compute the partial derivative of the cost function for each weight in the weight vector,

$$\Delta w_j = -\alpha \frac{\partial J}{\partial w_j}$$

Previous Lecture Overview

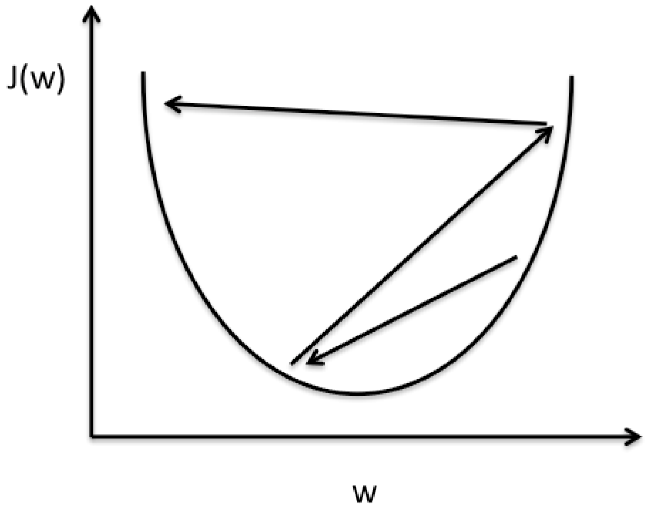
Adaptive Linear Neurons and the Delta Rule (1960)

- A step in gradient descent:
  - $\Delta w_j = -\alpha \frac{\partial J}{\partial w_j} = -\alpha \sum_i \left(y'^{(i)} - y^{(i)}\right)\left(-x_j^{(i)}\right) = \alpha \sum_i (y'^{(i)} - y^{(i)}) x_j^{(i)}$
- Update weight vector:
  - $\mathbf{w} := \mathbf{w} + \Delta\mathbf{w}$
- Differences with the perceptron rule
  - The output $y^{(i)}$ is a real number, not a class label as in perceptron learning rule.
  - Weight update is based on "all samples in the training set" (Batch GD)

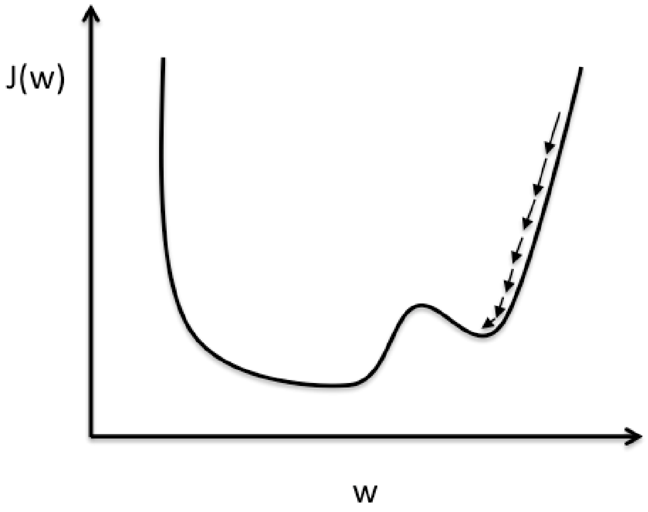
Previous Lecture Overview

Adaptive Linear Neurons and the Delta Rule (1960)

- If the learning rate is TOO LARGE, gradient descent will overshoot the minima and diverge.
- If the learning rate is too small, gradient descent will require too many epochs to converge and can become trapped in local minima more easily.



Large learning rate: Overshooting.



Small learning rate: Many iterations until convergence and trapping in local minima.

Adaptive Linear Neurons and the Delta Rule (1960)

- If features are scaled on the same scale, gradient descent converges faster and prevents weights from becoming too small (weight decay).

- Common way for feature scaling

$$x_{j,std} = \frac{x_j - \mu_j}{\sigma_j}$$

where μ_j is the sample mean of the feature x_j and σ_j the standard deviation.

- After standardization, the features will have unit variance and centered around mean zero.

Previous Lecture Overview

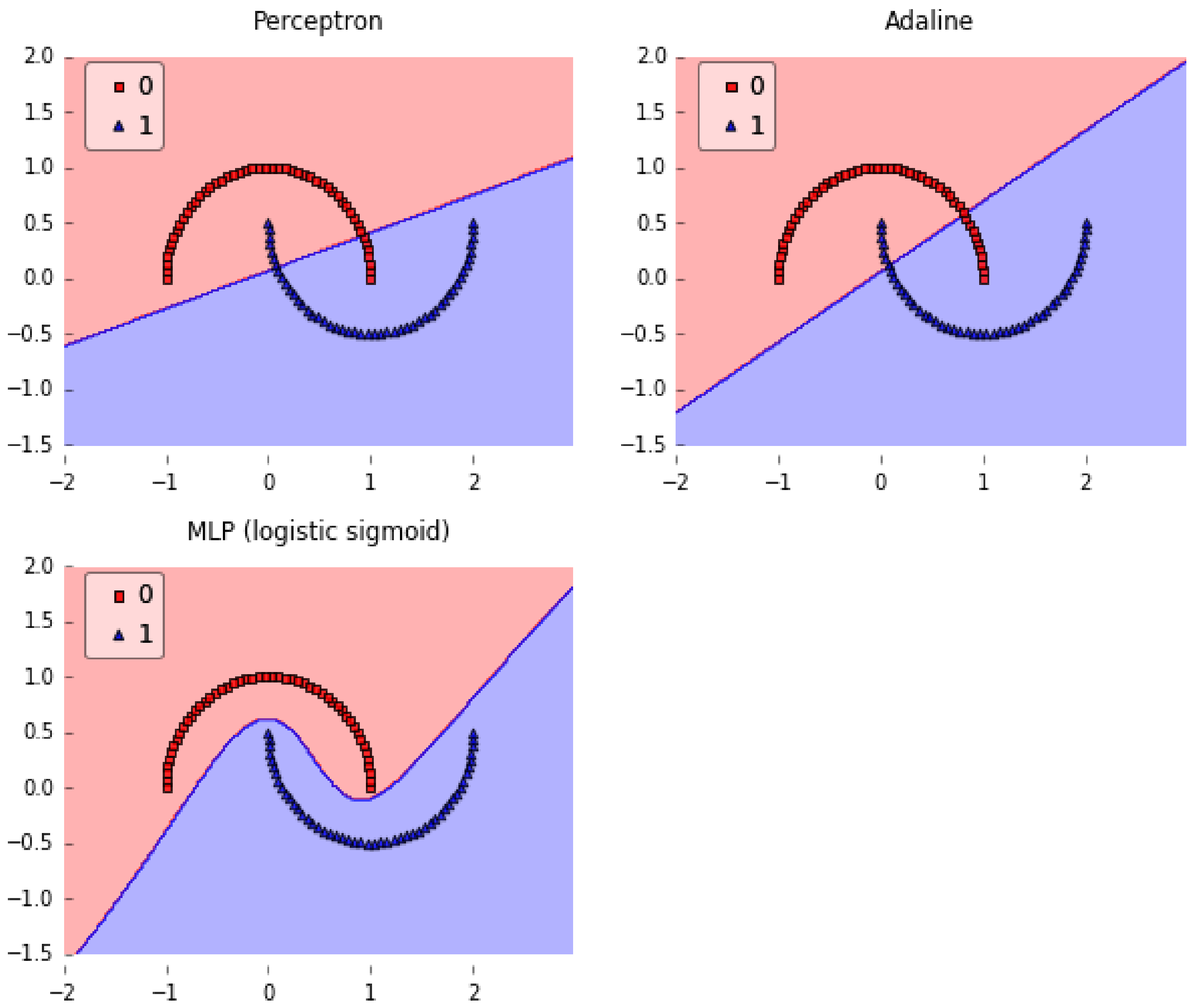
Adaptive Linear Neurons and the Delta Rule (1960)

- Batch Gradient Descent (BGD)
  - Cost function is minimized based on the complete training dataset (all samples)
- Stochastic Gradient Descent (SGD)
  - Weights are incrementally updated after each individual training sample
  - Converges faster than BGD since weights are updated immediately after each training sample
  - Computationally more efficient, especially for large datasets
- Mini-batch Gradient Descent (MGD)
  - Compromise between BGD and SGD, dataset is divided into mini-batches
  - Smoother convergence than SGD

Previous Lecture Overview

Logistic Regression

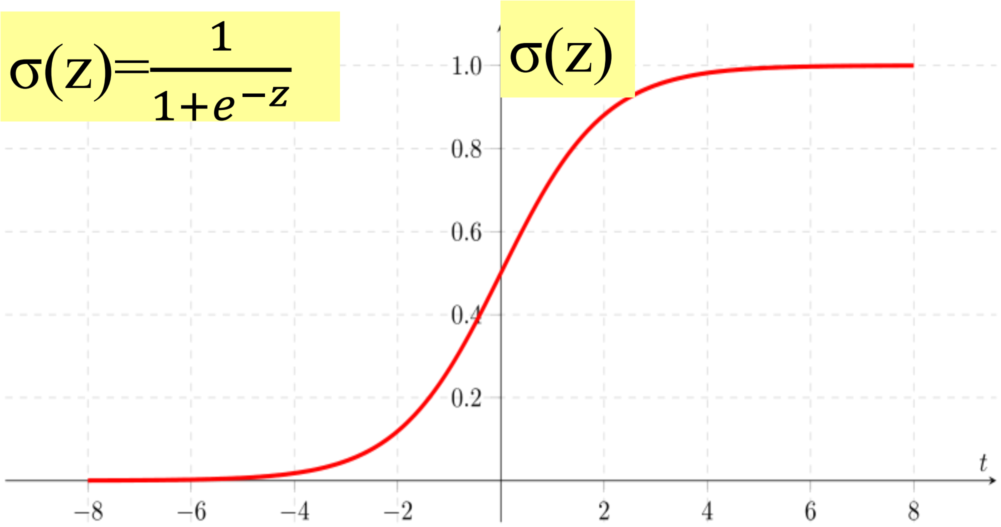
Perceptron vs. Adaline vs. Multi-Layer Perceptrons (Logistic Regression)



Previous Lecture Overview

Logistic Regression

$\sigma(z) = \frac{1}{1+e^{-z}}$



- Definition:
  - Given input $x \in \mathcal{R}^{n_x}$, calculate the probability $\hat{y} = P(y = 1|x), 0 \le \hat{y} \le 1.$
- Parameters:
  - Weights: $w \in \mathcal{R}^{n_x}$
  - Bias: $b \in \mathcal{R}$
- Output:
  - $\hat{y} = \sigma(z) = \sigma(w^T x + b)$ where $\sigma(z) = \frac{1}{1+e^{-z}}$ is the **sigmoid activation function**

If z is large positive number, σ(z) → 1

If z is small negative number, σ(z) → 0

Previous Lecture Overview

Logistic Regression

- Cost function is the average of all cross-entropy losses

$$\mathcal{J}(w,b) = \frac{1}{m}\sum_{i=1}^{m}\mathcal{L}(\hat{y}^{(i)}, y^{(i)}) = -\frac{1}{m}\sum_{i=1}^{m}\left[y^{(i)}\log\hat{y}^{(i)} + \left(1-y^{(i)}\right)\log\left(1-\hat{y}^{(i)}\right)\right]$$

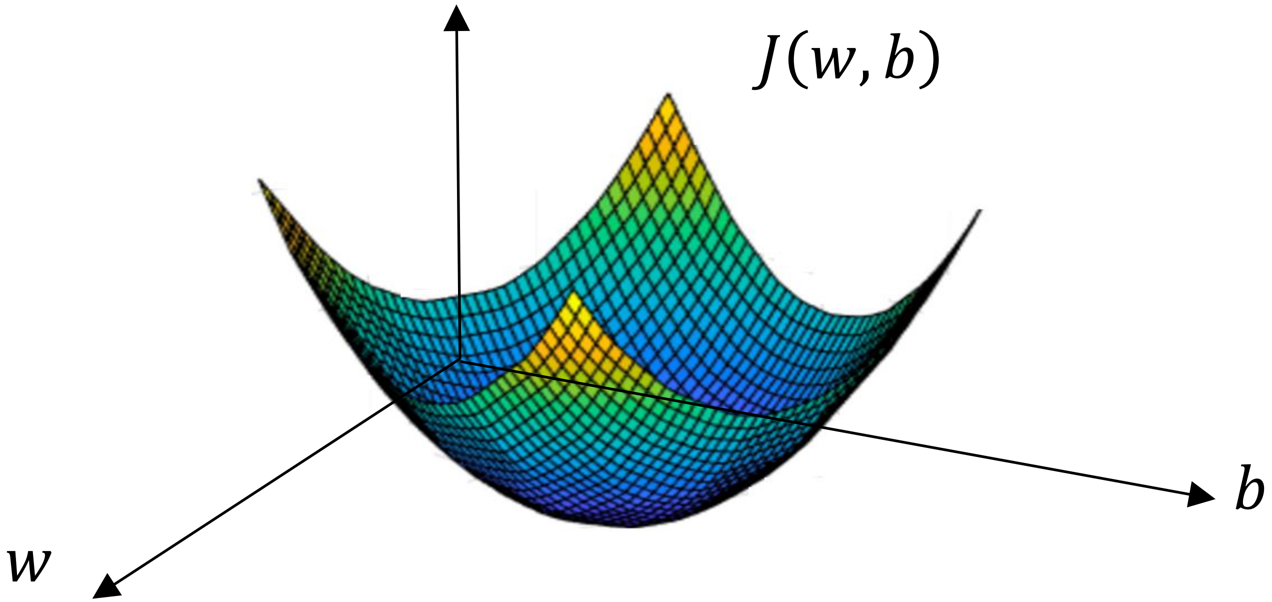
- Goal:
  - Find vectors **w** and **b** that minimize the cost function (total loss)
- Logistic regression can be viewed as a small neural network!

Previous Lecture Overview

Logistic Regression

Convergence

- $\hat{y}^{(i)} = \sigma\left(z^{(i)}\right) = \sigma\left(w^T x^{(i)} + b\right)$, where $\sigma\left(z^{(i)}\right) = \frac{1}{1+e^{-z^{(i)}}}$
- $\mathcal{J}(w, b) = \frac{1}{m}\sum_{i=1}^{m} \mathcal{L}(\hat{y}^{(i)}, y^{(i)}) = -\frac{1}{m}\sum_{i=1}^{m}\left[y^{(i)} \log \hat{y}^{(i)} + \left(1 - y^{(i)}\right) \log\left(1 - \hat{y}^{(i)}\right)\right]$
- Find w, b that minimize $\mathcal{J}(w, b)$

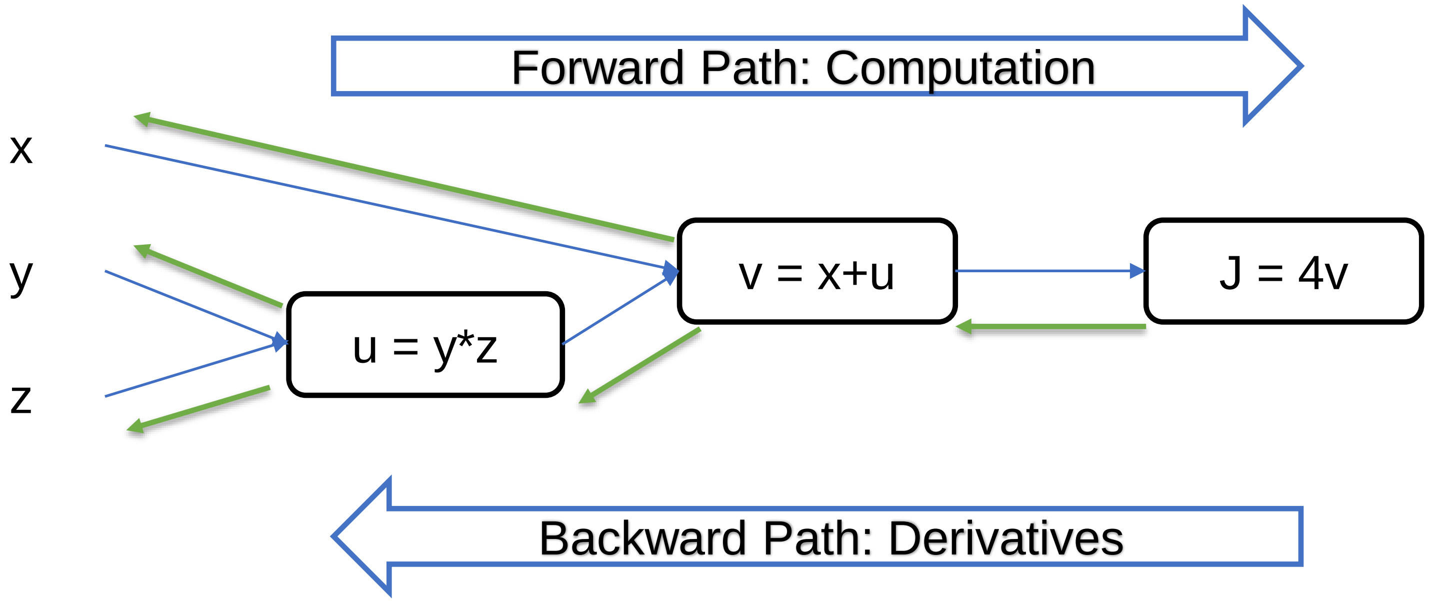


Previous Lecture Overview

Logistic Regression

Computation Graph

- A graph that depicts all the computations required for a function in a forward path
- For example: J(x, y, z) = 4(x + yz)



Previous Lecture Overview

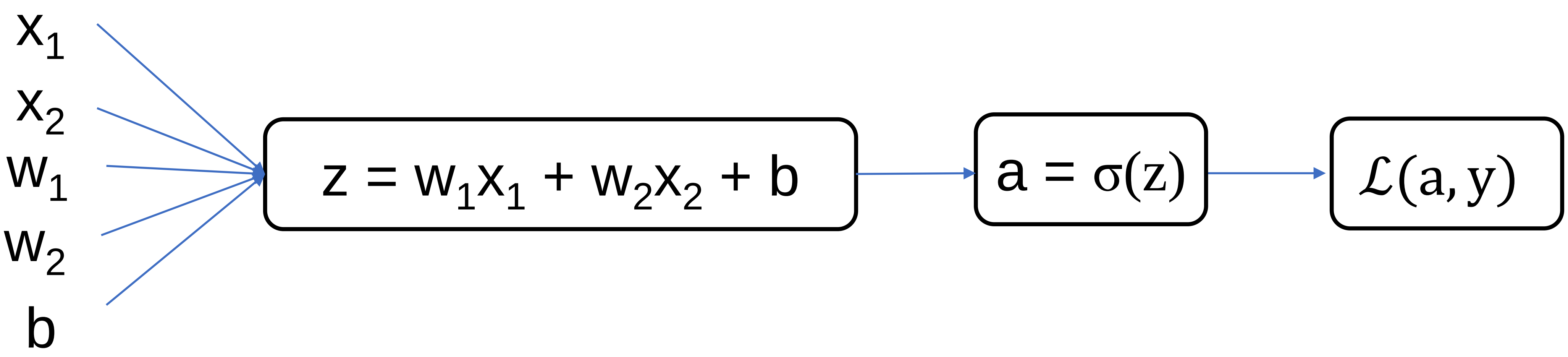
Logistic Regression

$$z = w^T x + b$$

$$\hat{y} = a = \sigma(z)$$

$$\mathcal{L}(a, y) = -(y \log(a) + (1 - y) \log(1 - a))$$

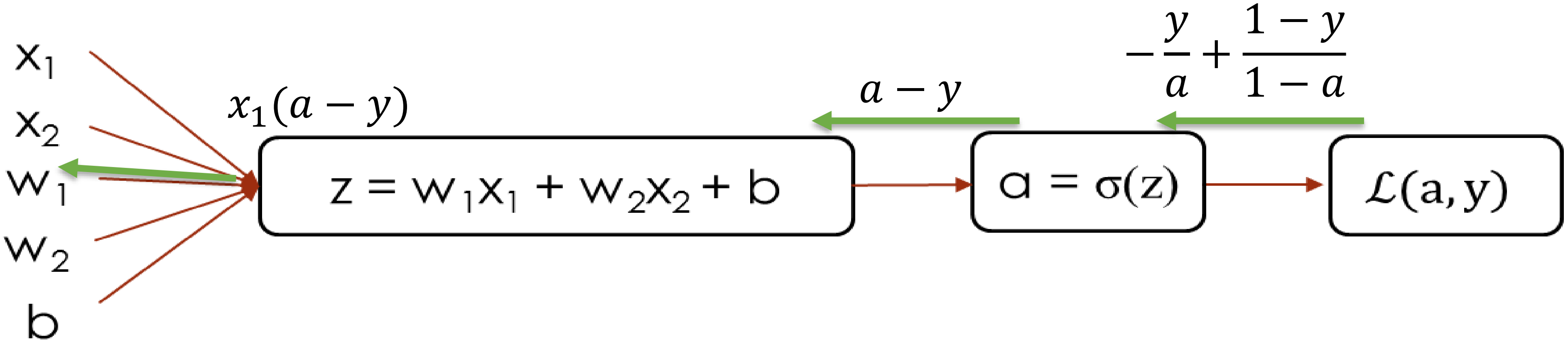
Computation Graph



Previous Lecture Overview

Logistic Regression

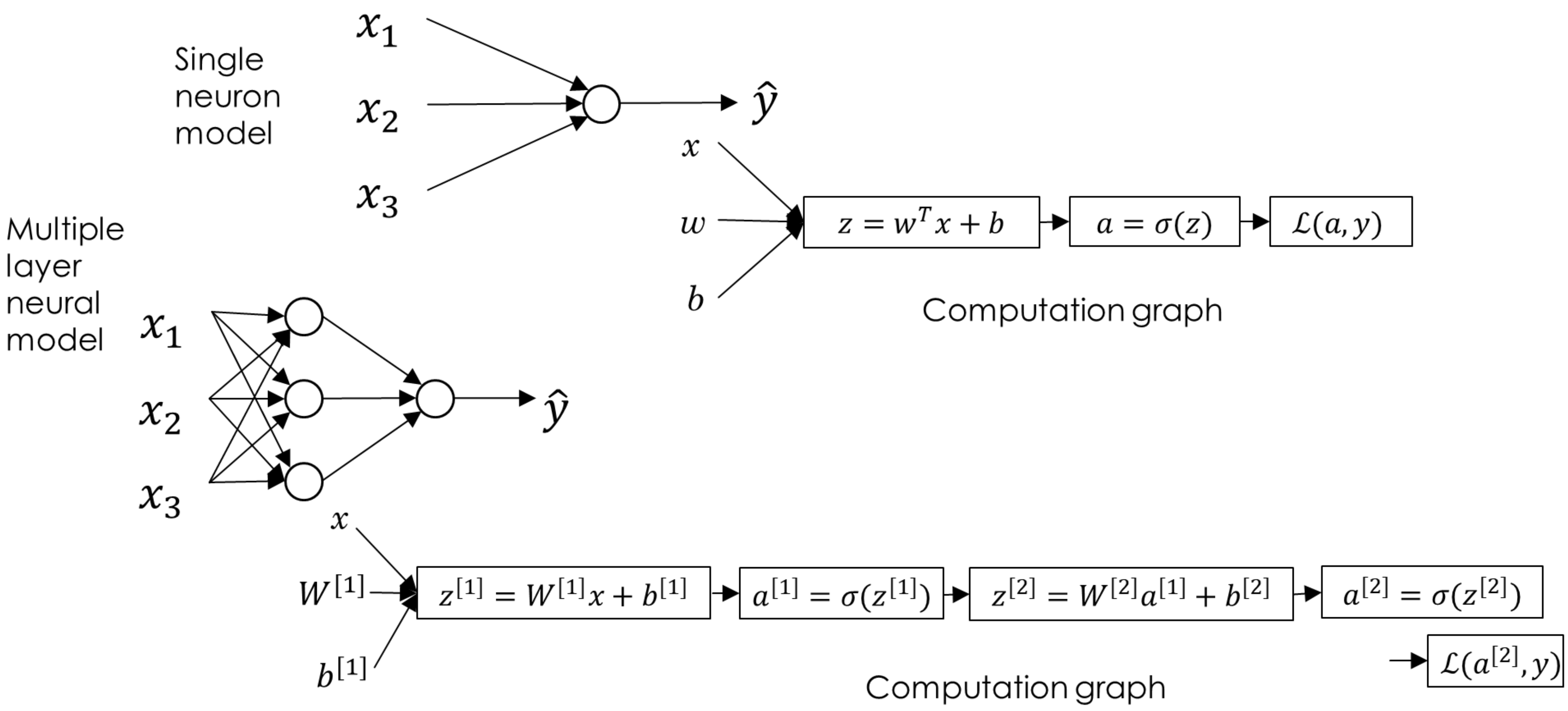
Computation Graph



- $\frac{\delta L(a,y)}{\delta a} = \left(-\frac{y}{a} + \frac{1-y}{1-a}\right)$
- $\frac{\delta L(a,y)}{\delta z} = \frac{\delta L(a,y)}{\delta a}\frac{\delta a}{\delta z} = \left(-\frac{y}{a} + \frac{1-y}{1-a}\right)\left(a(1-a)\right) = a - y$
- $\frac{\delta L(a,y)}{\delta w_1} = \frac{\delta L(a,y)}{\delta a}\frac{\delta a}{\delta z}\frac{\delta z}{\delta w_1} = \left(-\frac{y}{a} + \frac{1-y}{1-a}\right)a(1-a)x_1 = x_1(a-y) = x_1\frac{\delta L(a,y)}{\delta z}$

1. Shallow neural network

What is a Shallow Neural Network?

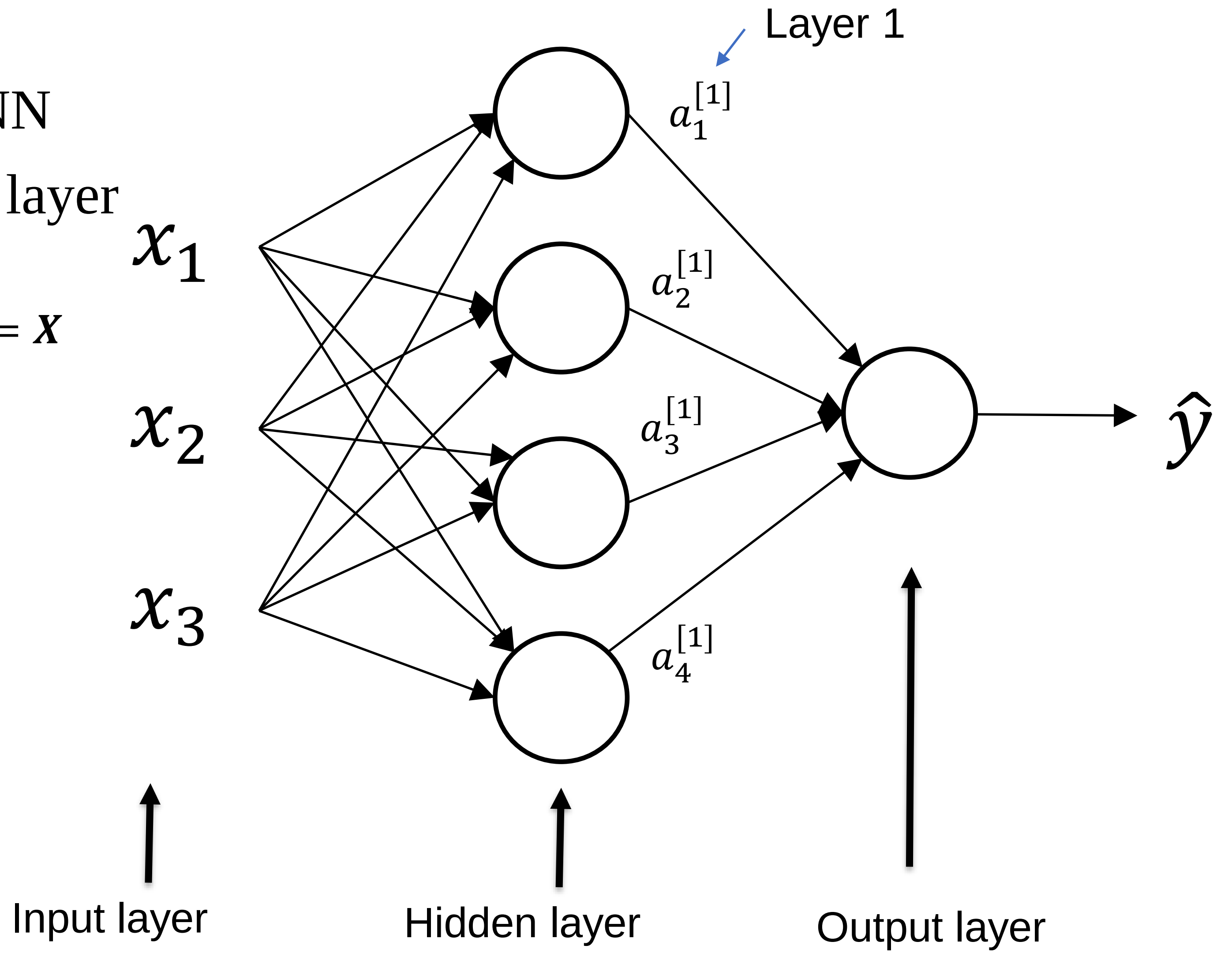


1. Shallow neural network

One hidden layer Neural Network

- 2-layer NN
- 1 hidden layer

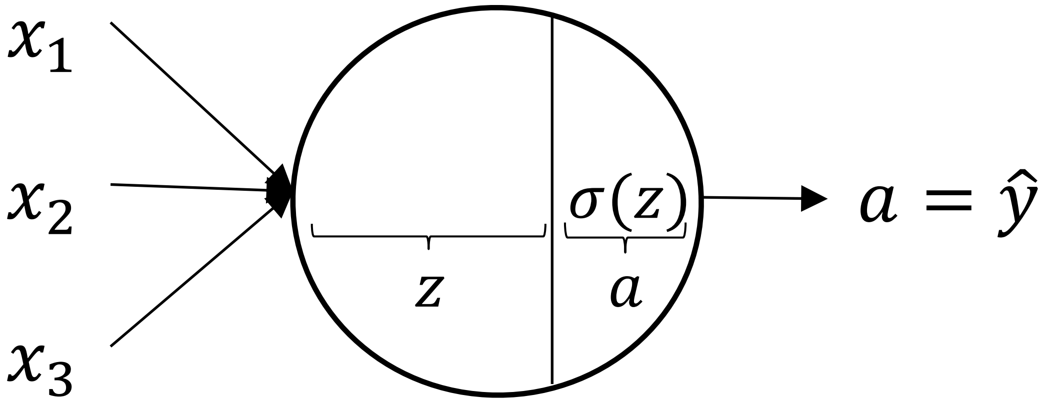
$a_0 = \boldsymbol{X}$



$= a^{[2]}$

$$a^{[1]} = \begin{bmatrix} a_1^{[1]} \\ a_2^{[1]} \\ a_3^{[1]} \\ a_4^{[1]} \end{bmatrix}$$

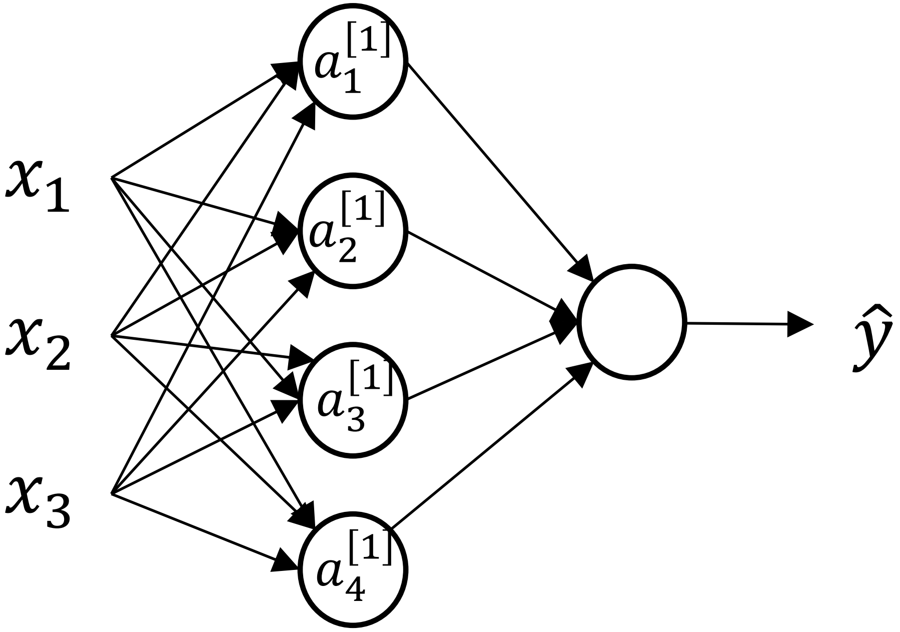
Computing NN's Output



$$z = w^T x + b$$

$$a = \sigma(z)$$

Computing NN's Output



$$z_1^{[1]} = w_1^{[1]T} x + b_1^{[1]},\ a_1^{[1]} = \sigma(z_1^{[1]})$$

$$z_2^{[1]} = w_2^{[1]T} x + b_2^{[1]},\ a_2^{[1]} = \sigma(z_2^{[1]})$$

$$z_3^{[1]} = w_3^{[1]T} x + b_3^{[1]},\ a_3^{[1]} = \sigma(z_3^{[1]})$$

$$z_4^{[1]} = w_4^{[1]T} x + b_4^{[1]},\ a_4^{[1]} = \sigma(z_4^{[1]})$$

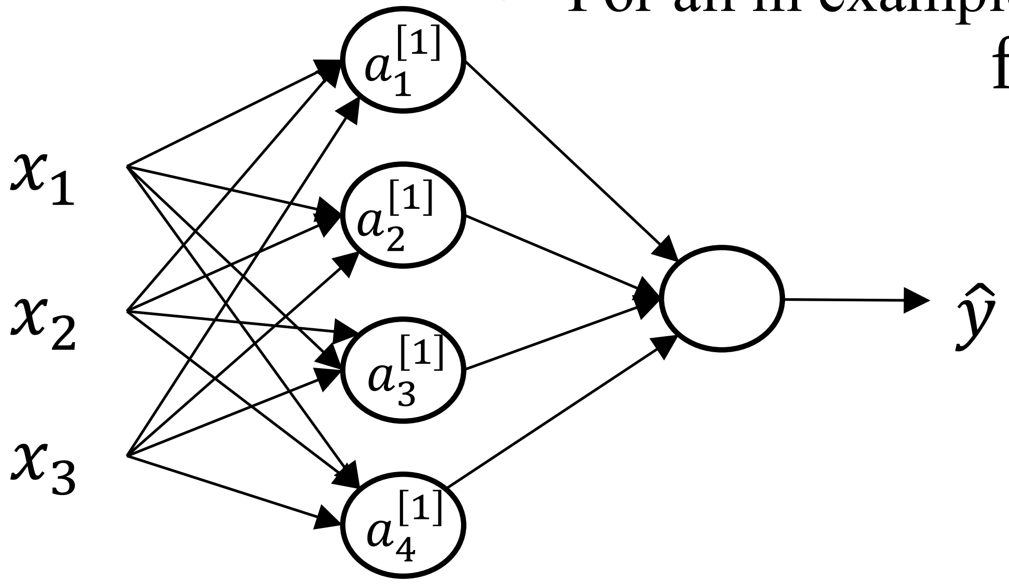
Given input x:

$$z^{[1]} = W^{[1]}x + b^{[1]}$$

$$z^{[2]} = W^{[2]}a^{[1]} + b^{[2]}$$

$$a^{[1]} = \sigma(z^{[1]})$$

$$a^{[2]} = \sigma(z^{[2]})$$



- For all m examples …

for i = 1 to m:

$$z^{[1](i)} = W^{[1]}x^{(i)} + b^{[1]}$$

$$a^{[1](i)} = \sigma(z^{[1](i)})$$

$$z^{[2](i)} = W^{[2]}a^{[1](i)} + b^{[2]}$$

$$a^{[2](i)} = \sigma(z^{[2](i)})$$

1. Shallow neural network

Vectorizing across multiple examples

$$X = \begin{bmatrix} | & | & & | \\ x^{(1)} & x^{(2)} & \dots & x^{(m)} \\ | & | & & | \end{bmatrix}$$

$$A^{[1]} = \begin{bmatrix} | & | & & | \\ a^{[1](1)} & a^{[1](2)} & \dots & a^{[1](m)} \\ | & | & & | \end{bmatrix}$$

```
for i = 1 to m:
```

$$z^{[1](i)} = W^{[1]}x^{(i)} + b^{[1]}$$

$$a^{[1](i)} = \sigma(z^{[1](i)})$$

$$z^{[2](i)} = W^{[2]}a^{[1](i)} + b^{[2]}$$

$$a^{[2](i)} = \sigma(z^{[2](i)})$$

$$Z^{[1]} = W^{[1]}X + b^{[1]}$$

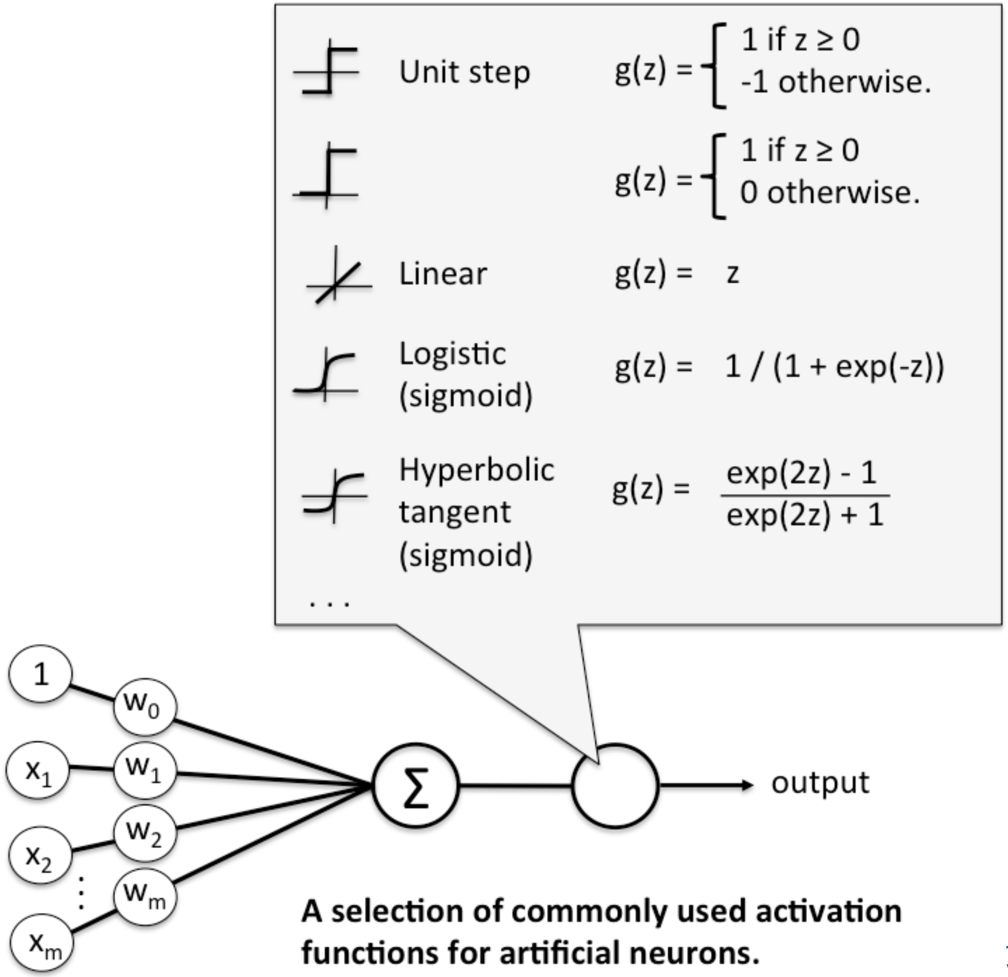
$$A^{[1]} = \sigma(Z^{[1]})$$

$$Z^{[2]} = W^{[2]}A^{[1]} + b^{[2]}$$

$$A^{[2]} = \sigma(Z^{[2]})$$

1. Shallow neural network

Activation functions



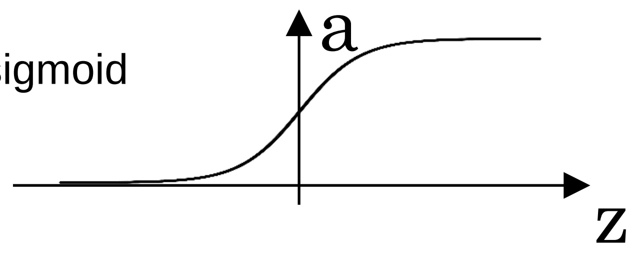
A selection of commonly used activation functions for artificial neurons.

Comprehensive List of Activation Functions: https://stats.stackexchange.com/questions/115258/comprehensive-list-of-activation-functions-in-neural-networks-with-pros-cons

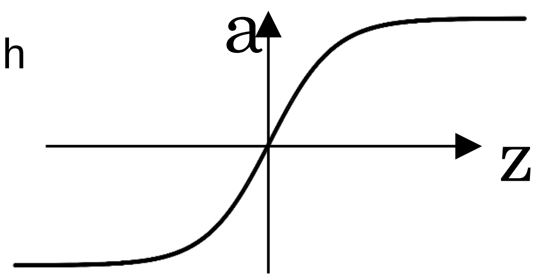
Activation functions

Activation Function	Formula (g(z))	Derivative (g'z))
sigmoid	$a = \frac{1}{1+e^{-z}}$	$a(1-a)$
tanh	$a = \frac{e^{z}-e^{-z}}{e^{z}+e^{-z}}$	$1-a^2$
ReLU	$\max(0, z)$	$0 \text{ if } z < 0$ $1 \text{ if } z \geq 0$
Leaky ReLU	$\max(0.01z, z)$	$0.01 \text{ if } z < 0$ $1 \text{ if } z \geq 0$

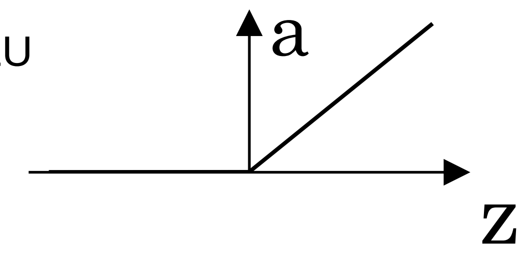
sigmoid



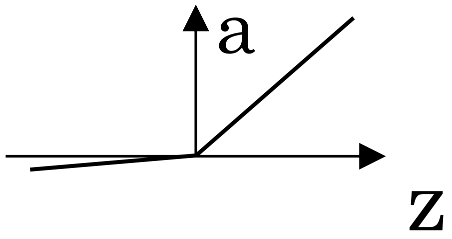
tanh



ReLU

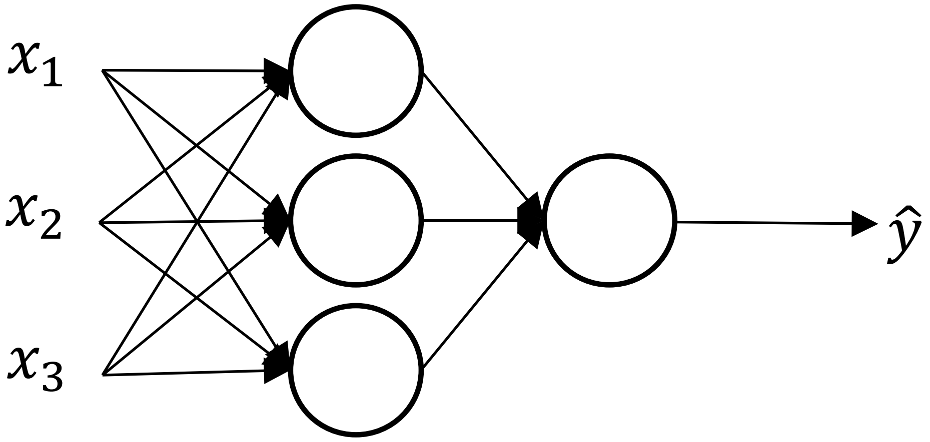


Leaky ReLU



1. Shallow neural network

Why non-linear activation function?



- What not linear?
- Suppose $g^{[1]}$, $g^{[2]}$ are all linear
  - $a^{[1]} = z^{[1]}$
  - $a^{[2]} = z^{[2]}$
  - $a^{[2]} = W^{[2]}a^{[1]}+b^{[2]}$
  - $= W^{[2]}(W^{[1]}X+b^{[1]})+b^{[2]}$
  - $= W^{[2]}W^{[1]}X + W^{[2]}b^{[1]}+b^{[2]}$
  - $= W'X + b'$
- All LINEAR!!!

Given x:

$$z^{[1]} = W^{[1]}x + b^{[1]}$$

$$a^{[1]} = g^{[1]}(z^{[1]})$$

$$z^{[2]} = W^{[2]}a^{[1]} + b^{[2]}$$

$$a^{[2]} = g^{[2]}(z^{[2]})$$

Gradient descent for one hidden layer

x, $W^{[1]}$, $b^{[1]}$

$$\boxed{z^{[1]} = W^{[1]}x + b^{[1]}} \rightarrow \boxed{a^{[1]} = \sigma(z^{[1]})}$$

$$dz^{[1]} = W^{[2]T}dz^{[2]} * g^{[1]\prime}(z^{[1]})$$

$$dW^{[1]} = dz^{[1]}x^T \qquad db^{[1]} = dz^{[1]}$$

$W^{[2]}$, $b^{[2]}$

$$\rightarrow \boxed{z^{[2]} = W^{[2]}x + b^{[2]}} \rightarrow \boxed{a^{[2]} = \sigma(z^{[2]})} \rightarrow \boxed{\mathcal{L}(a^{[2]}, y)}$$

$$dz^{[2]} = a^{[2]} - y$$

$$dW^{[2]} = dz^{[2]}{a^{[1]}}^T$$

$$db^{[2]} = dz^{[2]}$$

Vectorizing Gradient Descent

$$dz^{[2]} = a^{[2]} - y$$

$$dW^{[2]} = dz^{[2]}a^{[1]^T}$$

$$db^{[2]} = dz^{[2]}$$

$$dz^{[1]} = W^{[2]T}dz^{[2]} * g^{[1]\prime}(z^{[1]})$$

$$dW^{[1]} = dz^{[1]}x^T$$

$$db^{[1]} = dz^{[1]}$$

$$dZ^{[2]} = A^{[2]} - Y$$

$$J(..) = \frac{1}{m}\sum_{i=1}^{m}\mathcal{L}(\hat{y}, y)$$

$$dW^{[2]} = \frac{1}{m}dZ^{[2]}A^{[1]^T}$$

$$db^{[2]} = \frac{1}{m}np.sum(dZ^{[2]}, axis = 1, keepdims = True)$$

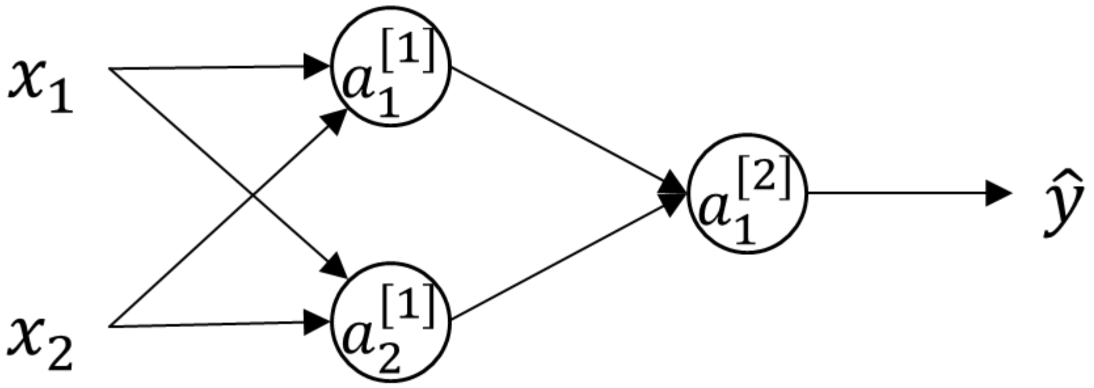
$$dZ^{[1]} = W^{[2]T}dZ^{[2]} * g^{[1]\prime}(Z^{[1]})$$

$$dW^{[1]} = \frac{1}{m}dZ^{[1]}X^T$$

$$db^{[1]} = \frac{1}{m}np.sum(dZ^{[1]}, axis = 1, keepdims = True)$$

1. Shallow neural network

Initializing weights



- What is weights are initialized to **zero**?
- Suppose all weights are zero:
  - $W^{[1]} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$
  - $b^{[1]} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$
  - $a_1^{[1]} = a_2^{[1]}$
  - $dz_1^{[1]} = dz_2^{[1]}$
  - $dW = \begin{bmatrix} u & v \\ u & v \end{bmatrix}$ (i.e., symmetric rows)
  - $W^{[1]} = W^{[1]} - \alpha dW$
  - $W^{[1]} = \begin{bmatrix} f & g \\ f & g \end{bmatrix}$ (i.e., symmetric rows)
- No need of TWO or more neurons … because all computations are same!
  - Do NOT initialize all weights are ZERO!!

1. Shallow neural network

Initialize weights RANDOMLY!

- W[1] = np.random.randn((2,2))***0.01**
  - **Small** random values are suggested!
  - If too large, $Z^{[1]} = W^{[1]}X + b^{[1]}$ will also be very large and $a^{[1]} = g^{[1]}(z^{[1]})$ will be in the flat areas and gradient descent will be very, very slooooooooow….

- b[1] = np.zero((2,1))   (b can be zero, no problem!)